GCE Examinations

Mechanics
Module M3

Advanced Subsidiary / Advanced Level

Paper A

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

When a numerical value of g is required, use $g = 9.8\ \text{m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1.

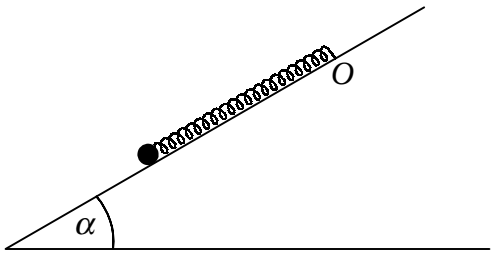

Fig. 1

A particle of mass 0.6 kg is attached to one end of a light elastic spring of natural length 1 m and modulus of elasticity 30 N. The other end of the spring is fixed to a point O which lies on a smooth plane inclined at an angle α to the horizontal where $\tan\alpha = \frac{3}{4}$ as shown in Figure 1.

The particle is held at rest on the slope at a point 1.2 m from O down the line of greatest slope of the plane.

(a) Find the tension in the spring. **(2 marks)**

(b) Find the initial acceleration of the particle. **(5 marks)**

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2. A particle P of mass 0.5 kg moves along the positive x-axis under the action of a single force directed away from the origin O. When P is x metres from O, the magnitude of the force is $3x^{\frac{1}{2}}$ N and P has a speed of v m s^{-1}.

Given that when $x = 1$, P is moving away from O with speed 2 m s^{-1},

(a) find an expression for v^2 in terms of x, **(5 marks)**

(b) show that when $x = 4$, P has a speed of 7.7 m s^{-1}, correct to 1 decimal place. **(2 marks)**

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3. A particle is performing simple harmonic motion along a straight line between the points A and B where $AB = 8$ m. The period of the motion is 12 seconds.

(a) Find the maximum speed of the particle in terms of π. **(4 marks)**

The points P and Q are on the line AB at distances of 3m and 6m respectively from A.

(b) Find, correct to 3 significant figures, the time it takes for the particle to travel directly from P to Q.

(6 marks)

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4. Whilst in free-fall a parachutist falls vertically such that his velocity, v m s^{-1}, when he is x metres below his initial position is given by

$$v^2 = kg(1 - e^{-\frac{2x}{k}}),$$

where k is a constant.

Given that he experiences an acceleration of f m s^{-2},

(a) show that $f = g e^{-\frac{2x}{k}}$. **(4 marks)**

After falling a large distance, his velocity is constant at 49 m s^{-1}.

(b) Find the value of k. **(3 marks)**

(c) Hence, express f in the form $(\lambda - \mu v^2)$ where λ and μ are constants which you should find. **(4 marks)**

5.

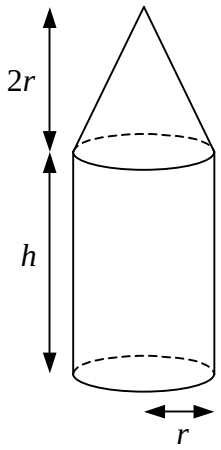

Fig. 2

A firework is modelled as a uniform solid formed by joining the plane surface of a right circular cone of height $2r$ and base radius r, to one of the plane surfaces of a cylinder of height h and base radius r as shown in Figure 2.

Using this model,

(a) show that the distance of the centre of mass of the firework from its plane base is

$$\frac{3h^2 + 4hr + 2r^2}{2(3h + 2r)}.$$ **(9 marks)**

The firework is to be launched from rough ground inclined at an angle α to the horizontal. Given that the firework does not slip or topple and that $h = 4r$,

(b) Find, correct to the nearest degree, the maximum value of α. **(4 marks)**

Turn over

6.

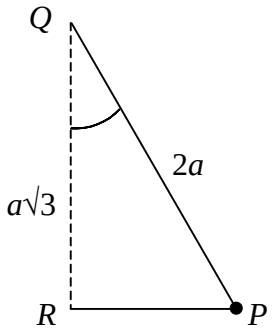

Fig. 3

The two ends of a light inextensible string of length $3a$ are attached to fixed points Q and R which are a distance of $a\sqrt{3}$ apart with R vertically below Q. A particle P of mass m is attached to the string at a distance of $2a$ from Q.

P is given a horizontal speed, u, such that it moves in a horizontal circle with both sections of the string taut as shown in Figure 3.

(a) Show that $\angle PRQ$ is a right angle. **(2 marks)**

(b) Find $\angle PQR$ in degrees. **(1 mark)**

(c) Find, in terms of a, g, m and u, the tension in the section of string

(i) PQ,

(ii) PR. **(7 marks)**

(d) Show that $u^2 \geq \frac{ga}{\sqrt{3}}$. **(3 marks)**

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7. A particle of mass 2 kg is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 50 N. The other end of the string is attached to a fixed point O on a rough horizontal plane and the coefficient of friction between the particle and the plane is $\frac{10}{49}$.

The particle is projected from O along the plane with an initial speed of $5\ \text{m s}^{-1}$.

(a) Show that the greatest distance from O which the particle reaches is 1.84 m. **(9 marks)**

(b) Find, correct to 2 significant figures, the speed at which the particle returns to O. **(5 marks)**

END